A nilpotent symmetry of quantum gauge theories

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Abstract

For the Becchi-Rouet-Stora-Tyutin (BRST) invariant extended action for any gauge theory, there exists another off-shell nilpotent symmetry. For linear gauges, it can be elevated to a symmetry of the quantum theory and used in the construction of the quantum effective action. Generalizations for nonlinear gauges and actions with higher order ghost terms are also possible.

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Introduction: Quantization of gauge theories requires gauge-fixing, and for most gauges, the introduction of ghost fields. The resulting theory is invariant, not under the gauge symmetry itself, but under the Becchi-Rouet-Stora-Tyutin (BRST) symmetry [1,2]. Nilpotence of the BRST transformation allows it to be extended to a symmetry of the quantum theory at all orders of the perturbation series, which allows order by order cancellation of infinities by the introduction of appropriate operators in the action. The quantum effective action is then the most general function invariant under this symmetry as well as all other known quantum symmetries of the theory.

In any useful gauge theory, gauge fields are coupled to many other fields. For general gauge theories, several of these fields may have the same Lorentz and gauge transformation properties. This leads to an enormous number of possible terms in the quantum effective action. If the theory is renormalizable, most of these terms have to vanish, leaving only those which are identical with the tree-level action, up to multiplicative constants. The BRST symmetry, imposed through the Slavnov-Taylor operator, ensures this stability for the physical ghost-free gauge-invariant part of the action. The demonstration of stability of the gauge-fixing and ghost terms requires auxiliary conditions.

In Landau-type gauges, the auxiliary conditions used are the ghost equation, the antighost equation, and their commutators with the Slavnov-Taylor operator [3]. In more general linear gauges the antighost equation picks up a nonlinear breaking term and thus loses its usefulness. This is particularly inconvenient for linear interpolating (R_ξ type) gauges. It is also inconvenient for gauge theories involving several fields with similar group and/or Lorentz transformation properties. For example, in theories involving non-Abelian two-form fields, one finds auxiliary vector fields and corresponding scalar ghosts. These can mix with the usual vector or ghost fields. As a result the proof of stability of general linear (including interpolating) gauges in such theories can be quite long. For nonlinear gauges and for actions containing terms of quartic or higher order in the ghost fields, proofs of stability are even more complicated.

In this paper I show that the BRST-invariant extended action for any gauge theory admits another, gauge fermion dependent, nilpotent symmetry, which does not seem to have been noticed earlier. This symmetry differs from the BRST symmetry only in its action on trivial pairs. For some special kinds of action, for example those which are quadratic in ghost fields, it becomes identical with the BRST symmetry upon using equations of motion. However, off shell it is always different from BRST, and can be used as an auxiliary condition to uniquely determine the quantum effective action of a gauge theory, including ghost and gauge-fixing terms. This symmetry holds in general linear gauges in addition to BRST, so it is particularly convenient for proving the uniqueness and stability of the ghost and gauge fixing sector of gauge theories outside Landau gauge, unlike the usual algebraic renormalization scheme. Below, I construct this symmetry, first when the theory has only fermionic ghosts, and then for a theory with both fermionic and bosonic ghosts. As a simple illustration I will apply this construction to the example of Yang-Mills theory, but the real convenience of this symmetry becomes apparent when it is applied to theories with larger field content, such as theories of p-form gauge fields. A generalization of the construction, somewhat similar to the well known antifield construction (see [4] for a review) suggests itself for theories with higher order ghost terms, and is discussed at the end of the paper.

The extended ghost sector of the tree-level quantum action of a gauge theory can be written in the general form

$$S^c_{ext} = h^A f^A + \frac{1}{2}\lambda h^A h^A + \bar{\omega}^A \Delta^A \,. \tag{1}$$

The anticommuting antighosts $\bar{\omega}^A$ and the corresponding auxiliary fields h^A form what are known as trivial pairs. Here the index A stands for the collection of all indices, $f^A = 0$ is the gauge-fixing condition, λ is a constant gauge-fixing parameter, and Δ^A is the BRST variation of the gauge-fixing function, $\Delta^A = sf^A$. The sum over A includes the integration over space-time, and f^A has been chosen to be independent of $\bar{\omega}^A$ and h^A. This part of the action remains invariant if the trivial pair transform under BRST as

$$s\bar{\omega}^A = -h^A \,, \qquad sh^A = 0 \,, \tag{2}$$

and can be written as a BRST differential of a gauge-fixing fermion Ψ,

$$S^c_{ext} = s\left(-\bar{\omega}^A f^A - \frac{1}{2}\lambda \bar{\omega}^A h^A\right) \equiv s\Psi \,. \tag{3}$$

On the other hand, I can rearrange S^c_{ext} as

$$\begin{aligned} S^c_{ext} &= \frac{1}{2}\lambda\left(h^A + \frac{1}{\lambda}f^A\right)\left(h^A + \frac{1}{\lambda}f^A\right) - \frac{1}{2\lambda}f^A f^A + \bar{\omega}^A\Delta^A \\ &= \frac{1}{2}\lambda\left(\left(h^A + \frac{2}{\lambda}f^A\right) - \frac{1}{\lambda}f^A\right)\left(\left(h^A + \frac{2}{\lambda}f^A\right) - \frac{1}{\lambda}f^A\right) - \frac{1}{2\lambda}f^A f^A + \bar{\omega}^A\Delta^A \\ &= h'^A f^A + \frac{1}{2}\lambda h'^A h'^A + \bar{\omega}^A \Delta^A \,, \end{aligned} \tag{4}$$

where I have defined $h'^A = -h^A - \dfrac{2}{\lambda}f^A$. Now S^c_{ext} has the same functional form as before, but in terms of a redefined auxiliary field h'^A. It follows that S^c_{ext} is invariant under a new set of transformations:

$$\begin{aligned} \tilde{s}\bar{\omega}^A = -h'^A &\Rightarrow \tilde{s}\bar{\omega}^A = h^A + \frac{2}{\lambda}f^A \,, \\ \tilde{s}h'^A = 0 &\Rightarrow \tilde{s}h^A = -\frac{2}{\lambda}\tilde{s}f^A \,, \\ \tilde{s} &= s \qquad \text{on all other fields.} \end{aligned} \tag{5}$$

It follows that $\tilde{s}$ is nilpotent on all fields, $\tilde{s}^2 = 0$. It should be emphasized that $\tilde{s}$ is a symmetry of the original (s-invariant) action itself, not some special feature of the construction procedure.

When the extended sector corresponds to the gauge-fixing of an anticommuting gauge field, as can happen for theories with reducible gauge symmetries, the construction is slightly more complicated, since the auxiliary fields now have odd ghost number. Typically, the extended ghost sector in this case can be written with anticommuting auxiliary fields $\bar{\alpha}^A$, α^A as

$$S^a_{ext} = \bar{\alpha}^A f'^A + \bar{f}'^A \alpha^A + \zeta \bar{\alpha}^A \alpha^A + \bar{\beta}^A \Delta'^A \,. \tag{6}$$

In this, f'^A is the anticommuting gauge-fixing function, $\Delta'^A = sf'^A$, and $\bar{\beta}^A$ is the corresponding commuting antighost. The term $\bar{f}'^A\alpha^A$ is a rearrangement of the appropriate terms in $\bar{\omega}^A\Delta^A$ which appear in S^c_{ext} of Eq. (1) for the usual gauge symmetries. Such terms are not affected by the redefinitions in Eq. (4), so they will appear in Eq. (6). In addition to Eq. (2), the BRST transformations on the extended sector now include $s\bar{\beta}^A = \bar{\alpha}^A, s\bar{\alpha}^A = s\alpha^A = 0$, and $s(S^c_{ext} + S^a_{ext}) = 0$, although S^c_{ext} and S^a_{ext} are not separately BRST-invariant.

Just as in the case with commuting auxiliary fields, the terms in S^a_{ext} can be rearranged,

$$\begin{aligned} S^a_{ext} &= \zeta\left(\bar{\alpha}^A + \frac{1}{\zeta}\bar{f}'^A\right)\left(\alpha^A + \frac{1}{\zeta}f'^A\right) - \frac{1}{\zeta}\bar{f}'^A f^A + \bar{\beta}^A\Delta'^A \\ &= \zeta\left(\left(\bar{\alpha}^A + \frac{2}{\zeta}\bar{f}'^A\right) - \frac{1}{\zeta}\bar{f}'^A\right)\left(\left(\alpha^A + \frac{2}{\zeta}f'^A\right) - \frac{1}{\zeta}f'^A\right) - \frac{1}{\zeta}\bar{f}'^A f^A + \bar{\beta}^A\Delta'^A \\ &= \zeta\bar{\alpha}'^A\alpha'^A + \bar{\alpha}'^A f'^A + \bar{f}'^A\alpha'^A + \bar{\beta}^A\Delta'^A\,. \end{aligned} \tag{7}$$

where I have now defined $\bar{\alpha}'^A = -\left(\bar{\alpha}^A + \frac{2}{\zeta}\bar{f}'^A\right)$ and $\alpha'^A = -\left(\alpha^A + \frac{2}{\zeta}f'^A\right)$. As before, a new set of BRST transformations can be defined for S^a_{ext},

$$\begin{aligned} \tilde{s}\bar{\beta}^A &= \bar{\alpha}'^A = -\left(\bar{\alpha}^A + \frac{2}{\zeta}\bar{f}'^A\right), \\ \tilde{s}\bar{\alpha}'^A &= 0 \Rightarrow \tilde{s}\bar{\alpha}^A = -\frac{2}{\zeta}\tilde{s}\bar{f}'^A\,, \\ \tilde{s}\alpha'^A &= 0 \Rightarrow \tilde{s}\alpha^A = -\frac{2}{\zeta}\tilde{s}f'^A\,, \\ \tilde{s} &= s \qquad \text{on all other fields}\,. \end{aligned} \tag{8}$$

Since α^A was the result of BRST variation of some field, α'^A has to be the variation under $\tilde{s}$ of the same field, and $\tilde{s}\bar{f}'^A$ must be calculated according to the rules of Eq. (5). In addition, the action of $\tilde{s}$ must be the same as that of s for the fields contained in f'^A. Then $\tilde{s}^2 = 0$ on all fields, and $\tilde{s}(S^c_{ext} + S^a_{ext}) = 0$.

Example: Let me consider a concrete example, and construct this symmetry for Yang-Mills theory in an arbitrary (linear or nonlinear) gauge-fixing function f^a. The tree-level quantum action is in this case

$$S = \int d^4x\left(-\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + h^a f^a + \bar{\omega}^a\Delta^a + \frac{1}{2}\xi h^a h^a\right), \tag{9}$$

where a is the gauge index. This is invariant under the BRST transformations

$$\begin{aligned} sA^a_\mu &= \partial_\mu\omega^a + gf^{abc}A^b_\mu\omega^c\,, \qquad s\bar{\omega}^a = -h^a\,, \\ s\omega^a &= -\frac{1}{2}gf^{abc}\omega^b\omega^c\,, \qquad sh^a = 0\,. \end{aligned} \tag{10}$$

Following the rules of Eq. (5), I obtain

$$\begin{aligned} \tilde{s}\bar{\omega}^a &= h^a + \frac{2}{\xi}f^a\,, \qquad \tilde{s}h^a = -\frac{2}{\xi}\Delta^a\,, \\ \tilde{s} &= s \text{ on all other fields.} \end{aligned} \tag{11}$$

By construction $\tilde{s}$ is a symmetry of the action, $\tilde{s}S = 0$, and nilpotent, $\tilde{s}^2 = 0$. It is straightforward to check these two properties explicitly for this example of Yang-Mills theory.

Any symmetry is a useful property of a theory, just how useful depends on both the symmetry and the theory. Let me show here how this symmetry can be used jointly with BRST to ease calculations. The quantum effective action $\Gamma[\chi, K]$ defined in the presence of background c-number sources K^A for the BRST variations $F^A = s\chi^A$ obeys the Zinn-Justin equation [5] $(\Gamma, \Gamma) = 0$, where $(\cdot, \cdot)$ is the antibracket in terms of χ^A and their sources for BRST variations, K^A. Note that $\Gamma[\chi, K]$ does not contain the sources for the BRST variations of auxiliary fields of the type $h^A, \alpha^A, \bar{\alpha}^A$, etc. which are BRST invariant. Also note that $\Gamma_{N,\infty}$, which is the infinite part of the N-loop contribution to Γ, does not contain the sources for the BRST variations of antighosts of the type $\bar{\omega}^A$, because their BRST variations are linear in the fields [6].

For most physically interesting cases the effective action is at most linear in the remaining K^A on dimensional grounds. This is the case for pure Yang-Mills fields, as well as several theories with Yang-Mills fields coupled to various other fields, in four dimensions. For these theories, the Zinn-Justin equation reduces to the statement that for infinitesimal ϵ, $S_R + \epsilon\Gamma_{N,\infty}$ is invariant under the quantum BRST symmetry s_R, which is just the most general nilpotent symmetry built out of the fields in the theory, and which reduces to the original BRST symmetry at tree-level [6].

Let me look at this class of theories, viz., those for which $\Gamma[\chi, K]$ has been shown to be at most linear in the K^A. Let me also assume that the quantum BRST transformation s_R has been found by solving the Zinn-Justin equation. In order to see the effect of the gauge-dependent symmetry $\tilde{s}$ on the quantum theory, I take the same effective action $\Gamma[\chi, K]$ with the same sources. Of course $\tilde{s}$ is a gauge-dependent symmetry, nonetheless it can be elevated to a symmetry of the quantum effective action if the gauge-fixing functions are linear in the fields. I shall denote the minimal fields by ϕ^A and non-minimal fields by λ^A. Then $\tilde{s}\phi^A = s\phi^A$, and consequently $\tilde{s}s\phi^A = 0$. The application of $\tilde{s}$ on the partition function gives (since the tree-level action S is invariant under $\tilde{s}$),

$$\langle F^A \rangle \frac{\delta_L \Gamma[\chi, K]}{\delta \phi^A} + \langle \tilde{s}\lambda^A \rangle \frac{\delta_L \Gamma[\chi, K]}{\delta \lambda^A} + \langle \tilde{s}s\lambda^A \rangle K^A[\lambda] = 0\,. \tag{12}$$

Here $\langle\ \rangle$ denotes the quantum average in the presence of sources, specified such that the quantum average of a field is the field itself [6]. So far the gauge could be arbitrary. For the special case where the gauge-fixing functions are assumed to be linear in the fields, $\tilde{s}\lambda^A$ as defined in Eq. (5) and Eq. (8) is either linear in the fields or equals the BRST variation of some linear function of the fields. Either way, $\langle \tilde{s}\lambda^A \rangle$ is known explicitly. In addition, the effective action does not contain the sources for BRST variations of $(h^A, \alpha^A, \bar{\alpha}^A)$ etc. and only S_R contains the sources for the BRST variations of $(\bar{\omega}^A, \bar{\beta}^A)$ etc. Then I can read off from Eq. (12) that $S_R + \epsilon\Gamma_{N,\infty}$ is invariant under $\tilde{s}_R$, which is just $\tilde{s}$ as calculated in terms of the quantum BRST transformation s_R.

Going back to the example of Yang-Mills theory in four dimensions, I obtain directly from Eq. (12) that in a linear gauge the quantum symmetry corresponding to $\tilde{s}$ is given just by Eq. (11) with s and $\tilde{s}$ replaced by s_R and $\tilde{s}_R$, respectively, where s_R is the usual quantum BRST transformation for Yang-Mills fields [5,6]. The ghost sector of the general effective action can now be obtained through an extremely short calculation. Let me define $s'_R = \frac{1}{2}(s_R - \tilde{s}_R)$. Then

$$s'_R\bar{\omega}^a = -\left(h^a + \frac{1}{\xi}f^a\right), \qquad s'_R h^a = \frac{1}{\xi}\Delta^a_R\,,$$
$$s'_R = 0 \text{ on all other fields.} \tag{13}$$

Since Yang-Mills theory is power-counting renormalizable in four dimensions, the infinite part of the N-loop quantum effective action, after infinities up to $N-1$ loops have been absorbed into counterterms, is an integrated local functional of mass dimension four. So on dimensional grounds and because the effective action must have zero ghost number, it can be at most quadratic in the trivial pair $\lambda^A \equiv (\bar{\omega}^a, h^a)$ [6]. So I can write

$$\Gamma = S_C + \lambda^A X^A + \lambda^A\lambda^B X^{AB}\,, \tag{14}$$

where S_C does not contain any ghost or auxiliary field, and X^A and X^{AB} do not contain any of the λ^A. Then the coefficients of different powers of the λ^A in the equation $s'_R\Gamma = 0$ must vanish. In particular, the terms quadratic or linear in λ^A give

$$\begin{gathered} X^{ab}_{\bar{\omega}\bar{\omega}} = X^{ab}_{\bar{\omega}h} = 0\,, \\ -X^a_{\bar{\omega}} + \frac{2}{\xi}\Delta^b_R X^{ab}_{hh} = 0, \end{gathered} \tag{15}$$

while the terms independent of λ^A in $s'_R\Gamma = 0$ give

$$-f^a X^a_{\bar{\omega}} + \Delta^a_R X^a_h = 0\,. \tag{16}$$

In addition, antighosts and auxiliary fields transform among themselves under BRST, so I can also consider the coefficients of λ^A in $s_R\Gamma = 0$ to obtain some independent equations,

$$s_R X^a_{\bar{\omega}} = 0 = s_R X^{ab}_{hh}, \qquad X^a_{\bar{\omega}} = s_R X^a_h\,. \tag{17}$$

Here the function X^{ab}_{hh} is symmetric and has vanishing mass dimension and ghost number. It follows from the above equation that X^{ab}_{hh} is purely numerical, and because we are dealing with the SU(N) algebra, and X^{ab}_{hh} is clearly symmetric in (a, b), it must be proportional to δ^{ab}. Then

$$X^{ab}_{hh} = \frac{\xi Z_\omega}{2}\delta^{ab}, \quad X^a_{\bar{\omega}} = Z_\omega \Delta^a_R \quad \text{and} \quad X^a_h = Z_\omega f^a\,, \tag{18}$$

for some constant Z_ω. (The last equation follows from combining Eq.s (17) and (16).) Therefore the quantum effective action takes the form

$$\Gamma = S_C + Z_\omega \bar{\omega}^a \Delta^a_R + Z_\omega h^a f^a + \frac{\xi}{2} Z_\omega h^a h^a\,, \tag{19}$$

where S_C is the most general ghost-free polynomial of dimension four symmetric under s_R and all linear symmetries of the classical theory. Note that I did not need to assume any specific gauge-fixing function, only that it is linear.

It is known that the problem of stability of the ghost (and gauge-fixing) sector of gauge theories can be solved by using the ghost and antighost equations as auxiliary conditions in the usual algebraic renormalization scheme [3]. However, those equations are in their most useful form in the Landau gauge $\xi = 0$, while the symmetry $\tilde{s}$ is defined for a non-zero ξ and cannot even be constructed directly for $\xi = 0$. (Of course, the $\xi \to 0$ limit can be taken after the effective action has been found.) Therefore the symmetry $\tilde{s}$ is not a reformulation of the usual auxiliary conditions. In particular, the use of $\tilde{s}$ as an auxiliary condition in the algebraic renormalization scheme, as opposed to the ghost and antighost equations, can be thought of as being complementary to those auxiliary conditions. This symmetry is especially useful in dealing with Yang-Mills type theories with a large number of fields, for which various different interpolating linear gauges are allowed. Examples are theories with p-form fields, as in the first order formulation of Yang-Mills theory [7] or the topological mass generation mechanism. The technique described here provides a straightforward way of verifying the uniqueness of the gauge-fixing and ghost sector of such theories, as has been done in [8].

Generalizations: The calculations for the example were done assuming that the gauge condition is linear in the fields. This was mainly for convenience — just as for the usual BRST symmetry, results in linear gauges are easier to calculate and interpret. But even in nonlinear gauges, or for actions with quartic ghost terms, there is a corresponding nilpotent symmetry. Let me show the construction for Yang-Mills theory in four dimensions, generalizations to many other cases being fairly simple. First, the gauge-fixing fermion of Eq. (3) is generalized to include terms quadratic in the antighost, so that

$$\begin{aligned} S^c_{ext} = s\Psi &= s(-\bar{\omega}^a f^a_0 - \frac{1}{2}\xi\bar{\omega}^a h^a - \frac{1}{2}\bar{\omega}^a\bar{\omega}^b f^{ab}_1) \\ &= \bar{\omega}^a \Delta^a_0 + h^a f^a_0 + \frac{1}{2}\xi h^a h^a + h^a\bar{\omega}^b f^{ab}_1 - \frac{1}{2}\bar{\omega}^a\bar{\omega}^b\Delta^{ab}_1\,, \end{aligned} \tag{20}$$

where f^a_0 and f^{ab}_1 do not contain $\bar{\omega}^a$ or h^a, but are arbitrary otherwise, and $\Delta^a_0 = sf^a_0$ and $\Delta^{ab}_1 = sf^{ab}_1$. For Yang-Mills theory in four dimensions, Ψ must be of dimension three or less, so there are no further terms. Now I can 'complete the square' as before, and write

$$S^c_{ext} = \frac{1}{2}\xi\left(h^a + \frac{1}{\xi}f^a\right)\left(h^a + \frac{1}{\xi}f^a\right) - \frac{1}{2\xi}f^a f^a + \bar{\omega}^a\Delta^a_0 - \frac{1}{2}\bar{\omega}^a\bar{\omega}^b\Delta^{ab}_1\,, \tag{21}$$

where $f^a = f^a_0 + \bar{\omega}^b f^{ab}_1$. Then as before I can define $h'^a = -\left(h^a + \frac{2}{\xi}f^a\right)$ and write

$$S^c_{ext} = \bar{\omega}^a\Delta^a_0 + h'^a f^a_0 + \frac{1}{2}\xi h'^a h'^a + h'^a\bar{\omega}^b f^{ab}_1 - \frac{1}{2}\bar{\omega}^a\bar{\omega}^b\Delta^{ab}_1\,, \tag{22}$$

which has the same functional form as Eq. (20), but with h^a replaced by h'^a. So the new symmetry transformations are

$$\begin{aligned} \tilde{s}\bar{\omega}^a &= h^a + \frac{2}{\xi}f^a_0 + \frac{2}{\xi}\bar{\omega}^b f^{ab}_1\,, \\ \tilde{s}h^a &= -\frac{2}{\xi}\Delta^a_0 - \frac{2}{\xi}h^b f^{ab}_1 - \frac{4}{\xi^2}f^b_0 f^{ab}_1 - \frac{4}{\xi^2}\bar{\omega}^c f^{bc}_1 f^{ab}_1 + \frac{2}{\xi}\bar{\omega}^b\Delta^{ab}_1\,, \\ \tilde{s} &= s \qquad \text{on all other fields.} \end{aligned} \tag{23}$$

Again, this is a symmetry of the action S^c_{ext}, and therefore a symmetry of the full action including the gauge-invariant terms. Note that $\tilde{s}$ is again nilpotent by construction, $\tilde{s}^2 = 0$.

For the examples given so far, including the last one, $\tilde{s}$ differs from s by a 'trivial symmetry' [9],

$$\tilde{s} - s = \eta^{AB} \frac{\delta S}{\delta \chi^A} \frac{\delta}{\delta \chi^B} \,, \tag{24}$$

where in this case χ^A is restricted to run over $(\bar{\omega}^A, h^A)$ and η^{AB} is graded antisymmetric in (A, B). What makes $\tilde{s}$ special is the fact that it is nilpotent, since adding a trivial symmetry to BRST does not make an off-shell nilpotent symmetry in general. On the other hand, for general BRST-invariant actions, the two symmetries s and $\tilde{s}$ need not be related by a trivial symmetry. For general actions, i.e., those which may include higher order ghost terms, the construction of $\tilde{s}$ can be generalized to give a nilpotent symmetry. To see how that can be done, note that for the examples above, $h'^A = \delta\Psi/\delta\bar{\omega}^A$ up to a constant coefficient, as if h'^A were the antifield of $\bar{\omega}^A$. It is worth emphasizing that h'^A is not the antifield for $\bar{\omega}^A$. But this similarity suggests a generalization of the previous constructions in the following way.

Given a gauge invariant action S_0, let the ghost fields be defined as usual, and introduce a trivial pair $(\bar{\omega}^A, h^A)$ for each generator, with the BRST transformation law $s\bar{\omega}^A = -h^A$, $sh^A = 0$. The gauge-fixing fermion Ψ is then constructed as some functional of ghost number -1, subject to any other known symmetry or dimensional restriction. The ghost sector of the action is then $s\Psi$, so that the total action $S_0 + s\Psi$ is BRST-invariant. Now let a new BRST transformation $\tilde{s}$ be defined as $\tilde{s}\bar{\omega}^A = -h'^A$, $\tilde{s}h'^A = 0$, where $h'^A = \delta\Psi/\delta\bar{\omega}^A$, and $\tilde{s} = s$ on all other fields. A new gauge fixing fermion Ψ' is then constructed by replacing h^A by h'^A in Ψ, i.e., $\Psi' = \Psi[h^A \to h'^A]$, and a new ghost action is constructed as $\tilde{s}\Psi'$. The (new) total action $S_0 + \tilde{s}\Psi'$ is then invariant under $\tilde{s}$.

If Ψ is chosen to be the most general gauge fixing fermion, $s\Psi$ would be the most general s-exact functional of vanishing ghost number. But now there are two actions, one constructed with $s\Psi$, and the other with $\tilde{s}\Psi'$, and these two need not be equal when written in terms of the same h^A. For the situations where (as in all the examples above) $\tilde{s}\Psi[h^A \to h'^A] = s\Psi$ up to a finite number of irrelevant constants, the total action is invariant under two different off-shell nilpotent symmetries s and $\tilde{s}$. This can be an immensely useful property for proving the uniqueness of the ghost action for complicated theories. In addition, since $\tilde{s}$ differs from BRST transformations only by its action on the trivial pair, it has the same cohomology as the BRST transformation itself [4]. So there is no additional complication in calculating the structure of anomalies in the theory, which is determined fully by the BRST cohomology.

In summary, any BRST invariant action in linear or nonlinear gauge has another off-shell, nilpotent symmetry with the same cohomology as the BRST transformation. If the action contains up to quartic ghost terms, it is always symmetric under both BRST and this transformation. If it contains higher order ghost terms, one can construct another action which is symmetric under the new BRST transformation, and whose gauge-invariant component agrees with that of the original action. If the ghost sectors of the two actions agree as well, both transformations leave it invariant. This can simplify calculations of the counterterms, especially when the gauge-fixing term is linear in the fields.

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